Method for Identifying PMSM Parameters

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This technical article presents a simple method for using Hioki's Power Analyzer to identify motor parameters (L_{d}, L_q, K_e, etc.) that are necessary when implementing vector control of permanent magnet synchronous motors.

1. Introduction

Permanent magnet synchronous motors (PMSMs) are widely used in the field of power electronics due to their high efficiency, high power density, and low weight. Depending on the location of the permanent magnet on the rotor, PMSMs can be classified as either interior permanent magnet synchronous motors (IPMSMs) or surface permanent magnet synchronous motors (SPMSMs) [1]. Due to their ability to use not only the magnetic torque of the permanent magnet, but also inductance torque [2], the range of applications of IPMSMs, which embed the permanent magnet inside the rotor, has grown to include EVs, aircraft, and inverter-powered household appliances [3] [4].

Typically, an equivalent circuit model [5] for the motor expressed in terms of the permanent magnet's N polar axis (d-axis) and the torque axis that is perpendicular to it (q-axis) is used when analyzing the characteristics of a PMSM and considering which control algorithm to use.

Eq. (1.1) defines the output torque T for this equivalent circuit model.

$$\mathrm{T} = P_n \phi_a i_q + P_n (L_d - L_q) i_q i_d \qquad (1.1)$$

Here, P_n represents the number of poles of the motor; i_d and i_q, the d- and q-axis components of each phase's armature current; $\mathrm{F_a}$, the RMS value for the permanent magnet's armature interlinkage magnetic flux; and L_d and L_q, the self-inductance of the d- and q-axes. The first term on the right side of eq. (1.1) indicates the magnetic torque, while the second term on the right side indicates the reluctance torque. Because SPMSMs have constant magnetic resistance regardless of their rotor position, $L_d = L_q$ is true in eq. (1.1), and the output torque consists entirely of magnetic torque. By contrast, there is a difference in the d-axis and q-axis inductance in IPMSMs for structural reasons ($L_d \neq L_q$), causing the reluctance torque to play a part in determining the output torque. Consequently, in order to maximize the output torque of an IPMSM, it is extremely important to identify the motor parameters that serve as constants in the equivalent circuit model (the inductance values L_d and L_q in the direction of the d- and q-axes) with a high degree of precision so that the reluctance torque can be controlled [6].

2. Need for Identifying PMSM Parameters Using a Power Analyzer

Inductance measurement using an LCR meter would appear to provide a simple method for identifying the motor parameters L_d and L_q [7]. However, that method suffers from the problem that it can only be used to identify motor parameters while the motor terminals are open and the motor is in the stopped state; it does not allow identification of motor parameters in the operating state. L_d and L_q include magnetic saturation characteristics, and as variables they incorporate a variety of dependencies that take into consideration current and other factors. Consequently, in order to realize high-precision control of a PMSM, it is necessary to identify L_d and L_q in a state of actual operation.

This article addresses this problem by introducing a simple yet high-precision method for identifying motor parameters in an operating state using the Power Analyzer.

3. Identification Principles

The output torque expressed in eq. (1.1) is based on the equation for a PMSM's voltage on the d-q coordinate axis. If we assume the following, the equation for a PMSM's voltage expressed on the d-q coordinate axis can be expressed as indicated in (3.1) below [5].

i) The spatial distribution of magnetic flux in the gap between the stator and rotor takes the form of a sine wave running along the gap.
ii) Voltage and current harmonic components can be ignored.
iii) Iron (core) loss can be ignored.

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R + pL_d & -\omega L_q \\ \omega L_d & R + pL_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \omega \phi_a \end{bmatrix} \quad (3.1)$$

Here, v_d and v_q represent the d- and q -axis components of each phase's armature current; R, each phase's armature resistance; p, the differential operator (d/dt); ω, the rotation angle (electric angle) speed; and F_a ($=K_e$), the RMS value of the permanent magnet's armature interlinkage magnetic flux (induced voltage constant). If we assume a steady state (by ignoring the time derivative term) and express eq. (3.1) as a vector diagram for the d- and q-axes, the result is Fig. 3.1.

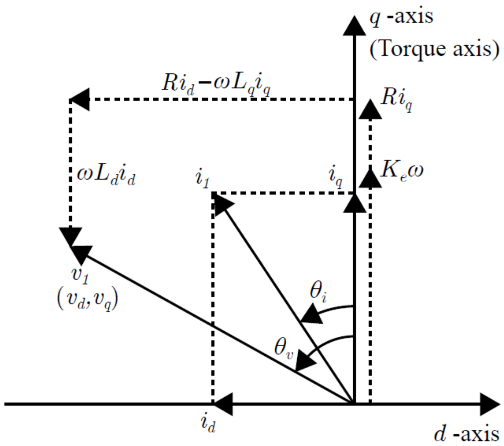


Fig. 3.1 PMSM vector diagram

Here, v_1 and i_1 represent the fundamental wave components of the phase voltage and phase current, respectively, while q_v and q_i represent the fundamental wave phase angle for the phase voltage and phase current, respectively. Based on Fig. 3.1, the voltage equations in the d- and q-axis directions are as follows:

$$K_e\omega + Ri_q = v_q - \omega L_d i_d \quad (3.2)$$

$$v_d = Ri_d - \omega L_q i_q \quad (3.3)$$

Solving for L_d and L_q yields:

$$L_d = \frac{v_q - K_e\omega - Ri_q}{\omega i_d} \quad (3.4)$$

$$L_q = \frac{Ri_d - v_d}{\omega i_q} \quad (3.5)$$

4. Conversion from Symmetric Three-Phase AC to the *dq* Coordinate System

This section describes the derivation process for converting symmetric three-phase AC to the $\alpha\beta$ coordinate system. Note that this assumes the conditions i), ii), and iii) in Chapter 3 are satisfied. First, consider the conversion from symmetric three-phase AC to the α-β coordinate system (Clarke transformation) (Fig. 4.1). When expressing the three-phase currents i_u, i_v, and i_w in α-β coordinates where the U-phase current aligns with the α-axis, the currents i_α and i_β are given by the following matrix equation.

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{\sqrt{2}}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_u \\ i_v \\ i_w \end{bmatrix} \quad (4.1)$$

In this article, the coefficient for the conversion from symmetric three-phase AC to the $\alpha\beta$ coordinate system is set to $(\sqrt{2})/3$. This coefficient ensures that the RMS value of the three-phase vector before the Clarke transformation becomes the peak value after the $\alpha\beta$ coordinate transformation (hereinafter referred to as RMS transformation).
For absolute transformation, where instantaneous power remains invariant before and after conversion, the coefficient is $\sqrt{(2/3)}$. Additionally, for relative transformation, where the current amplitude remains invariant before and after conversion, the coefficient is 2/3.

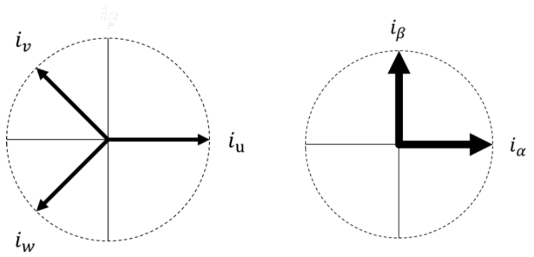


Fig. 4.1 Image of Clarke transformation

Next, the matrix equation for the Park transformation, where the coordinate advanced by θ[rad] from the $\alpha\beta$ coordinate is the d-q axis, is given by the following (Fig. 4.2).

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{4.2}$$

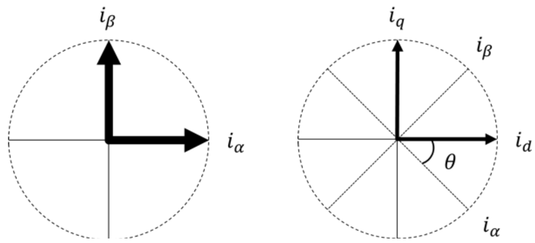


Fig. 4.2 Image of Park transformation

From eq. (4.1) and (4.2), the transformation equation from three-phase vector to i_d and i_q on the d-q axis is as follows.

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \frac{\sqrt{2}}{3} \begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2}{3}\pi\right) & \cos\left(\theta + \frac{2}{3}\pi\right) \\ -\sin\theta & -\sin\left(\theta - \frac{2}{3}\pi\right) & -\sin\left(\theta + \frac{2}{3}\pi\right) \end{bmatrix} \begin{bmatrix} i_u \\ i_v \\ i_w \end{bmatrix} \tag{4.3}$$

Here, the three-phase AC current is expressed by the following equation. At this time, I is the RMS value of the phase current, and γ indicates that the U-phase current i_u is advanced by γ[rad] from the d-axis.

$$\begin{bmatrix} i_u \\ i_v \\ i_w \end{bmatrix} = \sqrt{2}\, I \begin{bmatrix} \sin(\omega t + \gamma) \\ \sin\left(\omega t - \frac{2}{3}\pi + \gamma\right) \\ \sin\left(\omega t + \frac{2}{3}\pi + \gamma\right) \end{bmatrix} \tag{4.4}$$

In eq. (4.3) and (4.4), setting the advance rotation angle θ[rad] from the $\alpha\beta$ coordinate to ωt ($\theta = \omega t$) and organizing using the product-sum formula for trigonometric functions, i_d and i_q are obtained as follows.

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = I \begin{bmatrix} \sin\gamma \\ -\cos\gamma \end{bmatrix} \tag{4.5}$$

From eq. (4.5), currents can be treated as DC on the d-q axis (Fig. 4.3).

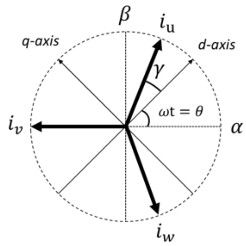


Fig. 4.3 Three-phase vector diagram (current)

Furthermore, if the advance rotation angle of the armature current vector from the q-axis is set to δ_i [rad], then $\gamma = \delta_i + \pi/2$, and substituting this into (4.5) yields the following equation.

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = A \cdot I \begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \end{bmatrix} \tag{4.6}$$

Here, A in the equation represents the transformation coefficient. For RMS transformation, the coefficient is 1; for absolute transformation, it is $\sqrt{3}$; and for relative transformation, it is $\sqrt{2}$.
Similarly, if the advance phase of the armature voltage vector from the q-axis is set to δ_v [rad], the voltages vd and v_q on the d-q coordinate axis are expressed using the RMS value V of the U-phase voltage by the following equation.

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = A \cdot V \begin{bmatrix} -\sin\delta_v \\ \cos\delta_v \end{bmatrix} \tag{4.7}$$

Similarly, for RMS transformation, the coefficient is 1; for absolute transformation, it is $\sqrt{3}$; and for relative transformation, it is $\sqrt{2}$.

Illustrating the above on the d-q coordinate axis results in Fig. 4.4. By setting the q-axis as the reference, with $\delta_i = \theta_i$ and $\delta_v = \theta_v$, it can be directly replaced with the vector diagram in Fig. 3.1.

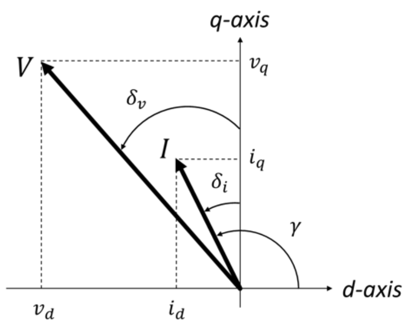


Fig. 4.4 $d - q$ *axis vector diagram*

In the case of RMS transformation, since the RMS value of the three-phase vector becomes the peak value after Clarke transformation, the following holds.

$$\sqrt{i_\alpha^2 + i_\beta^2} = \sqrt{i_d^2 + i_q^2} = I \tag{4.8}$$

$$\sqrt{v_\alpha^2 + v_\beta^2} = \sqrt{v_d^2 + v_q^2} = V \tag{4.9}$$

Therefore, i_d, i_q, v_d, and v_q can be directly obtained from the phase current and phase voltage RMS values measured by the power analyzer. Even if instantaneous noise is superimposed on the waveform, RMS values can be obtained as relatively stable values.
For reference, Table 1 shows the correspondence table of coefficients based on RMS transformation. When seeking values for absolute or relative transformation, they can be derived simply by multiplying the parameters calculated based on RMS transformation by the coefficient using the UDF (User-Defined Function) function.
Note that the self-inductance values L_d and L_q are constant regardless of the coefficient in the Clarke transformation.

Conversion type		RMS relative	Absolute	Relative
Clerk conversion Coefficient		$\frac{\sqrt{2}}{3}$	$\sqrt{\frac{2}{3}}$	$\frac{2}{3}$
Voltage/current		$\sqrt{i_d^2+i_q^2}=I$ $\sqrt{v_d^2+v_q^2}=V$	$\sqrt{i_d^2+i_q^2}=\sqrt{3}I$ $\sqrt{v_d^2+v_q^2}=\sqrt{3}V$	$\sqrt{i_d^2+i_q^2}=\sqrt{2}I$ $\sqrt{v_d^2+v_q^2}=\sqrt{2}V$
d-q axis	Voltage	v_d v_q	$\sqrt{3}v_d$ $\sqrt{3}v_q$	$\sqrt{2}v_d$ $\sqrt{2}v_q$
	Current	i_d i_q	$\sqrt{3}i_d$ $\sqrt{3}i_q$	$\sqrt{2}i_d$ $\sqrt{2}i_q$
	Inductance	L_d L_q	L_d L_q	L_d L_q

Table. 1 Correspondence Table of Coefficients in Clarke-Park Transformation

Additionally, the induced voltage constant K_e in RMS transformation is expressed by the following equation. Here, ω_m is the mechanical angular velocity [rad/s], p is the number of poles, and N is the rotation speed [rpm].

$$K_e = \frac{v_q}{\omega} = \frac{v_q}{\frac{p}{2}\cdot\omega_m} = \frac{v_q}{\frac{p}{2}\cdot 2\pi\frac{N}{60}} \tag{4.10}$$

If the induced voltage constant for absolute transformation is $K_{e\,abs}$ and for relative transformation is $K_{e\,rlt}$, then the following holds.

$$K_{e\,abs} = \sqrt{3}\cdot K_e \tag{4.11}$$
$$K_{e\,rlt} = \sqrt{2}\cdot K_e \tag{4.12}$$

Based on the preceding derivation of the Clarke and Park transformations, using the output torque eq. (1.1) in the equivalent model, the actual three-phase power and torque in RMS transformation are expressed as follows.

$$P_{uvw} = 3P_{dq} = 3\left(v_d i_d + v_q i_q\right) \tag{4.13}$$
$$T_{uvw} = 3T_{dq} = 3\cdot\frac{p}{2}\left(K_e i_q + \left(L_d - L_q\right)i_d i_q\right) \tag{4.14}$$

In the next chapter, we introduce the procedure for identifying PMSM parameters using actual measuring instruments.

5. Identification procedure

This section uses Hioki's Power Analyzer PW8001 as an example to describe motor parameter identification. Note that the calculation formulas are based on RMS transformation.

5.1Measurement of each phase's armature resistance *R*

First, use a resistance meter or other suitable instrument to measure each phase's armature resistance *R*.
For star connection, if the motor's neutral point cannot be accessed, measure the resistance between two phases and calculate it (Fig. 5.1).

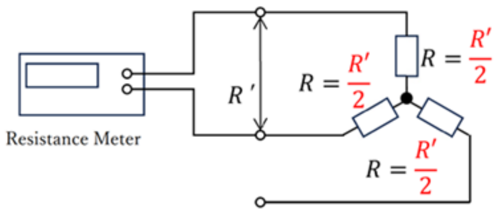
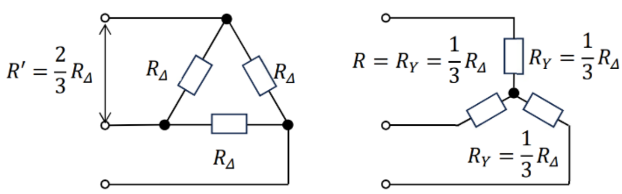


Fig. 5.1 Measurement of armature resistance in star connection

$$R = \frac{R'}{2} \tag{5.1}$$

Even for delta connection, the resistance value per stator phase (phase armature resistance after Δ-Y conversion) is half the measured resistance value (Fig. 5.2).



Measurement of armature resistance in delta connection

$$R = \frac{R'}{2} \tag{5.2}$$

5.2 Phase zero-adjustment and identification of the induced voltage constant K_e

After placing the terminals of the PMSM under measurement in the open state ($i_d = i_q = 0$), connect the motor terminals to the CH 1, 2, and 3 voltage inputs on the PW8001. Next, connect the encoder's A-phase pulse output to CH B (or CH F); the B-phase pulse output to CH C (or CH G); and the Z-phase pulse (origin signal) output to CH D (or CH H) (Fig. 5.3).

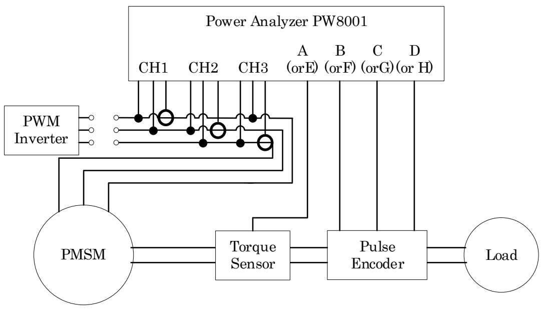


Fig. 5.3 Wiring for phase zero-adjustment and identification of the induced voltage constant

Configure the PW8001 by setting the motor analysis operation mode to "Single" and the measurement parameter to "Torque Speed Direction Origin."
Next, set the CH 1, 2, and 3 wiring type to 3P3W3M; the synchronization source and harmonic synchronization source to "Ext1"; and Δ conversion to "ON." Setting the measurement channel synchronization source and harmonic synchronization source to "Ext1" allows measurement of the voltage and current phase angle using the inputted encoder pulse as a reference, while setting Δ conversion to "ON" allows conversion of line voltage to phase voltage for measurement.

Drive the motor in this state from the load side to generate an induced voltage and perform phase zero-adjustment on the PW8001. Note that the rotation speed at this time should preferably be the same as when measuring the axis parameters later by rotating from the inverter side with load applied. Doing so will cause q_v and q_i to become the phase voltage (that is, electric angle) based on the induced voltage phase occurring in the q-axis direction.

At this time, an induced voltage ($v_q = v_1$) will occur, and eq. (3.4) will become:

$$K_e = \frac{v_q}{\omega} = \frac{v_1}{2\pi f_1} \tag{5.3}$$

The equation now allows identification of K_e. Here, f_1 (= ω / 2π) indicates the frequency of the phase voltage's fundamental wave.

5.3 Identification of the motor parameters L_d and L_q using user-defined functions

Self-inductance (L_d and L_q) in the direction of the d- and q-axes can be identified using R as measured in Section 5.1 and K_e as identified in Section 5.2. Connect the inverter's drive output to the motor terminals which were placed in an open state in Section 5.2 and operate the motor (Fig. 5.4).

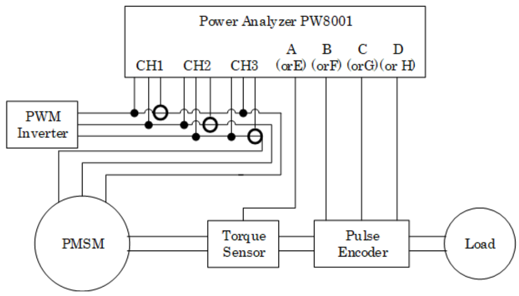


Fig. 5.4 Wiring when identifying motor parameters

Based on Fig. 3.1, the following will obtain at this time:

$$v_d = -v_1 \sin\theta_v \tag{5.4}$$

$$v_q = v_1 \cos\theta_v \tag{5.5}$$

$$i_d = -i_1 \sin\theta_i \tag{5.6}$$

$$i_q = i_1 \cos\theta_i \tag{5.7}$$

If these equations along with eq. (3.4) and (3.5) are configured as user-defined functions (UDFs), you can easily identify L_d and L_q while monitoring i_d and i_q.

Following are some specific example settings.
Following are some specific example settings. First, set UDF_1 and UDF_2 to v_d and v_q, respectively.

$$\mathrm{UDF}_1 = -\mathrm{U}_{\mathrm{fnd1}} \cdot \sin\theta_{\mathrm{U1}}$$

$$\mathrm{UDF}_2 = \mathrm{U}_{\mathrm{fnd1}} \cdot \cos\theta_{\mathrm{U1}}$$

Here, U_{fnd1} and θ_{U1} are notations for the basic measurement items of CH1 on the PW8001, representing the fundamental wave component (the primary AC signal) of the voltage RMS value and the voltage phase angle, respectively, and the following calculation formula holds.

$$\mathrm{U_{fnd1}} = U_1$$

$$\theta_{\mathrm{U1}} = \theta_v$$

Next, set $\mathrm{UDF_3}$ and $\mathrm{UDF_4}$ to i_d and i_q.

$$\mathrm{UDF_3} = -\mathrm{I_{fnd1}} \cdot \sin\theta_{\mathrm{I1}}$$

$$\mathrm{UDF_4} = \mathrm{I_{fnd1}} \cdot \cos\theta_{\mathrm{I1}}$$

Here $\mathrm{I_{fnd1}}$ and q_{I1} represent the basic measurement parameters for CH 1 on the PW8001, indicating the fundamental wave component of the current RMS value and the current phase angle, respectively, as follows:

$$\mathrm{I_{fnd1}} = i_1$$

$$\theta_{\mathrm{I1}} = \theta_i$$

Next, set L_d. The numerator of eq. (3.4) is as follows:

$$\mathrm{UDF_5} = \mathrm{U_{fnd1}} \cdot \cos\theta_{\mathrm{U1}} - (2\pi K_e) \cdot f_1 \\ -R \cdot \mathrm{UDF_4}$$

The denominator of eq. (3.4) is as follows:

$$\mathrm{UDF_6} = (2\pi) \cdot f_1 \cdot \mathrm{UDF_3}$$

Consequently, L_d can be expressed as follows:

$$\mathrm{UDF_7} = \mathrm{UDF_5}/\mathrm{UDF_6}$$

Finally, set L_q. The numerator of eq. (3.5) is as follows:

$$\mathrm{UDF_8} = R \cdot \mathrm{UDF_3} - (-v_{\mathrm{fnd1}}) \cdot \sin\theta_{\mathrm{U1}}$$

The denominator of eq. (3.5) is as follows:

$$\mathrm{UDF_9} = (2\pi) \cdot f_1 \cdot \mathrm{UDF_4}$$

Consequently, L_q can be expressed as follows:

$$\mathrm{UDF_{10}} = \mathrm{UDF_8}/\mathrm{UDF_9}$$

Fig. 5.5, 5.6 and 5.7 depict the UDF settings screen on the Power Analyzer PW8001 when $\mathrm{UDF_{1\text{-}10}}$ have been configured in this way. In Fig. 5.6 and 5.7, R = 1.2 [Ω] and K_e = 20 [mV×s/rad], and the second and third terms on the right side of $\mathrm{UDF_5}$ as well as the second term on the right side of $\mathrm{UDF_{10}}$ have been set accordingly.

6. Conclusion

This article introduces a simple yet high-precision method for identifying motor parameters in an operating state using the Power Analyzer. Please see other resources [8] and [9] that introduce how to use a Hioki Power Analyzer PW6001 to identify motor parameters along with actual measurement results. Note that the method introduced in this article yields motor parameters for an equivalent circuit model that assumes the circuit is in a steady state and that iron (core) loss can be ignored.

Since the identification method introduced in this article makes it comparatively easy to measure the current dependency of the motor parameters L_d and L_q, it can be used to create tools like L_d and L_q maps and torque maps while the motor is in an operating state in order to implement optimal control of PMSMs.

References

[1] Control-use Electromagnetic Actuator Drive System Investigation Committee: "Control-use Electromagnetic Actuator Drive Systems," Technical Reports of The Institute of Electrical Engineers of Japan, No. 719, pp.1-8 (1993).
[2] Y. Takeda, N. Matsumoto, S. Morimoto and Y. Honda: Interior Permanent Magnet Synchronous Motor Design and Control, Ohmsha (2011).
[3] Y. Hori: "Future Vehicle driven by Electricity and Control - Research on 4 Wheel Motored 'UOT March II,'" IEEE Trans. on Industry Electronics, Vol. 51, No. 5, pp.954.462 (2004).
[4] P. Alvarez, M. Satrustegui, I. Elosegui and M. Martinez-Iturralde: "Review of High Power and High Voltage Electric Motors for Single-Aisle Regional Aircraft," IEEE Access, Vol. 10, pp.112989-113004 (2022).
[5] S. Morimoto, Y. Takeda and T. Hirasa: "Method for Measuring the Constants of a PM Motor dq Equivalent Circuit," Transactions of The Institute of Electrical Engineers of Japan D, Vol. 113-D, No. 11,

pp.1330-1331 (1993).
[6] H. Domeki: Capable Field Tools: Electric Motor Drive Basics and Applications, Kato Bunmeisha (2010).
[7] M. Shogo, K. Masanobu, T. Yoshiyasu, T. Mamoru, S. Toru, O. Motomichi, H. Tsuyoshi and A. Takashi: "Development of a Stator-Magnetless Linear Synchronous Motor for Sensorless Control," IEEE Transactions on Industry Applications, Vol. 53, No. 5, pp.4559-4568 (2017).
[8] HIOKI E.E. Corp.: "Identification of PMSM Parameters with the Power Analyzer PW6001" (White Paper) (2016).
[9] HIOKI E.E. Corp.: "Identification of PMSM Motor Parameters with a Power Analyzer" (White Paper) (2016).

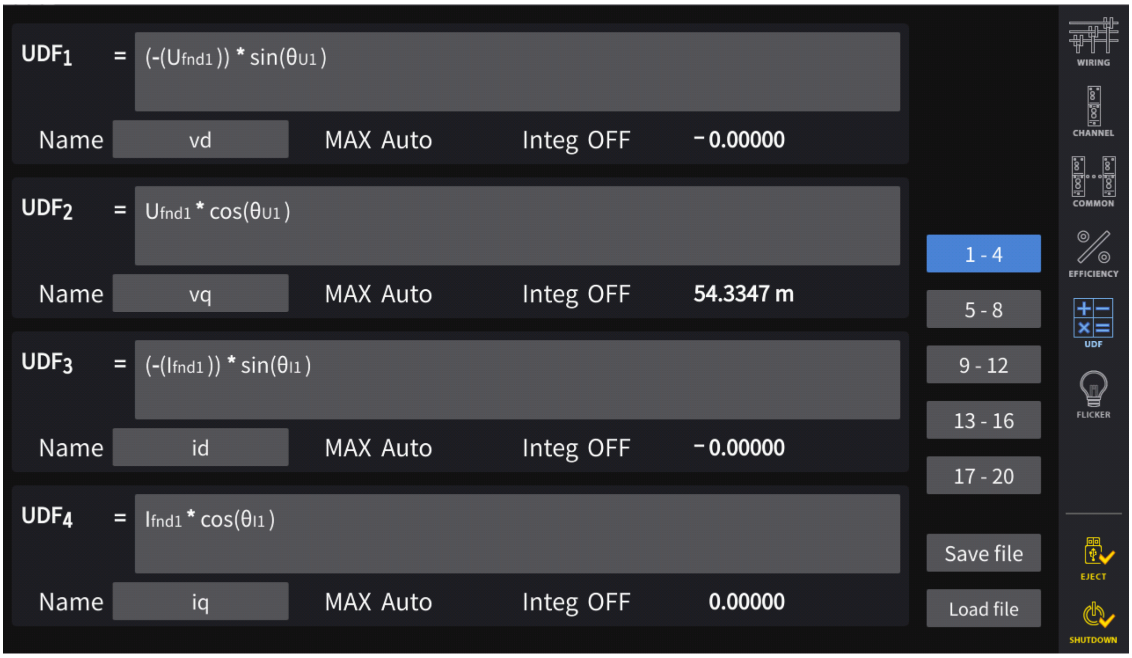


Fig. 5.5 Example PW8001 UDF settings ($UDF_{1\text{-}4}$)

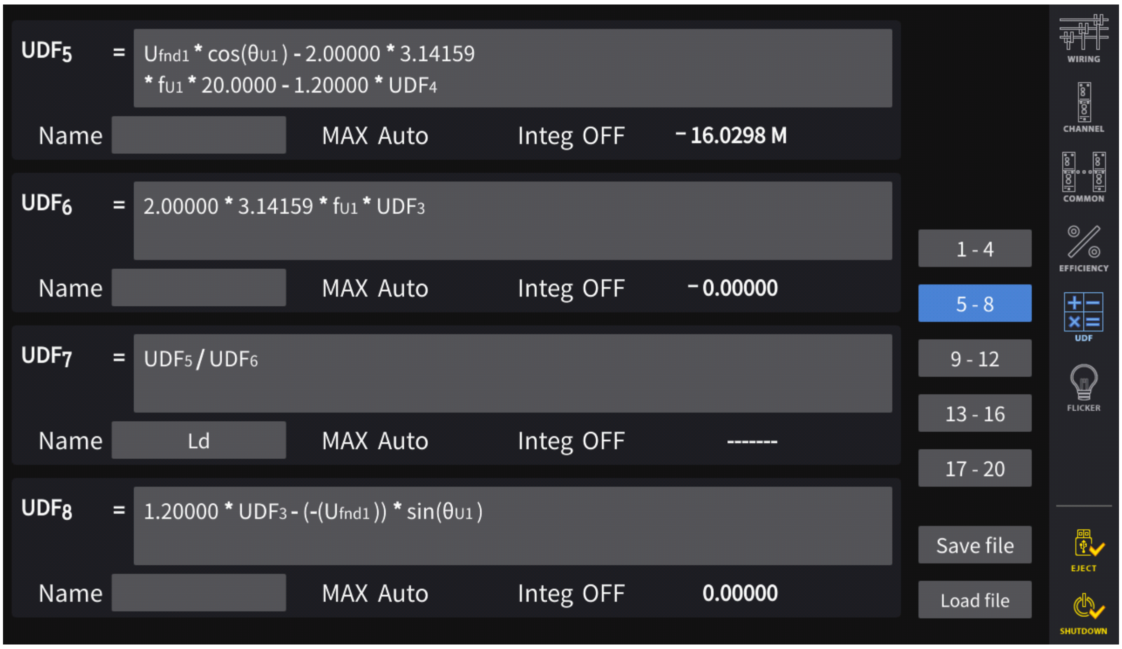


Fig. 5.6 Example PW8001 UDF settings ($UDF_{5\text{-}8}$)

Fig. 5.7 Example PW8001 UDF settings ($UDF_{9\text{-}10}$)